

Rock slope failure along non-persistent joints – insights from fracture mechanics approach

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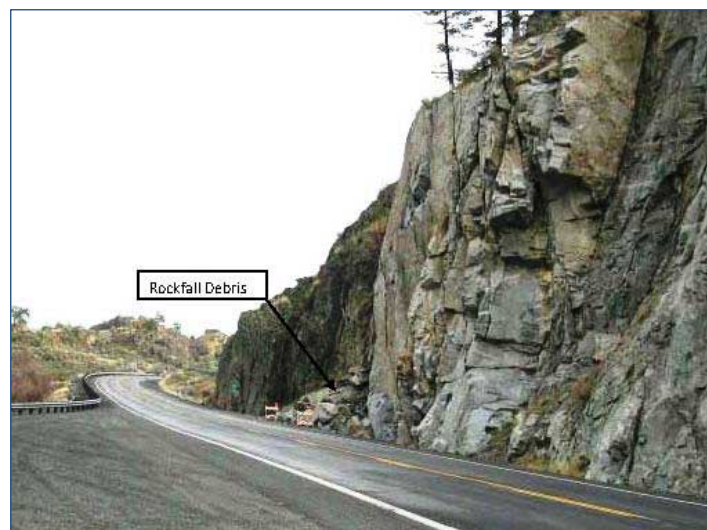
Nanyang Technological University, Singapore



Slope Stability

First Principle of Slope Engineering

All slopes are potentially unstable.



Stability is controlled by

- Material type and properties of rock slope
- Geological structures (faults, joints, etc)
- Geometry of slope
- External factors (weather, water, seismic events, blasting, excavation, etc)
- Rock slopes are predominantly **structurally controlled** (discontinuities)

<http://www.geo-design.co.uk/wp-content/uploads/2010/06/Ting-Kau-Bridge-Approach-Road-Rock-Slopes-Hong-Kong.jpg>



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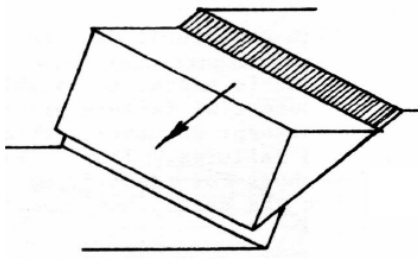
Tennessee Rock Slope Failure



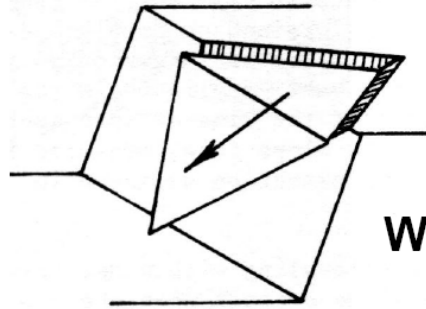
http://www.youtube.com/watch?v=ZVYGJYnJTi0&feature=player_detailpage

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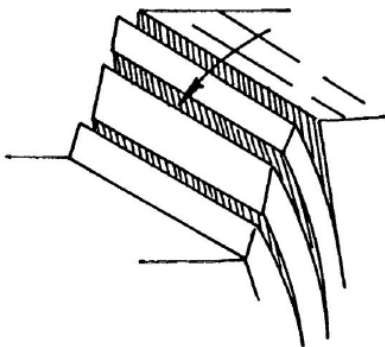
Failure Modes of Rock Slopes



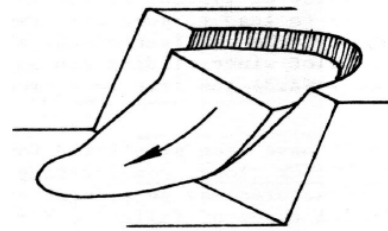
Plane failure



Wedge failure



Toppling failure



Circular failure

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Plane (Planar) rock slope failure



Sliding of a rock mass along a **relatively planar** failure surface.

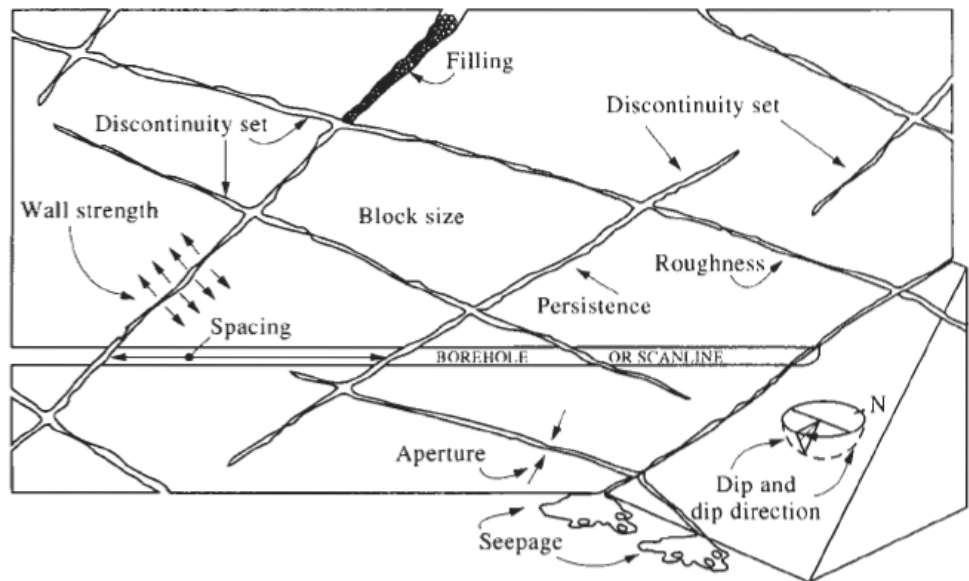
Why meta-stable?

- Shear strength of rock joint
- Rock bridge strength

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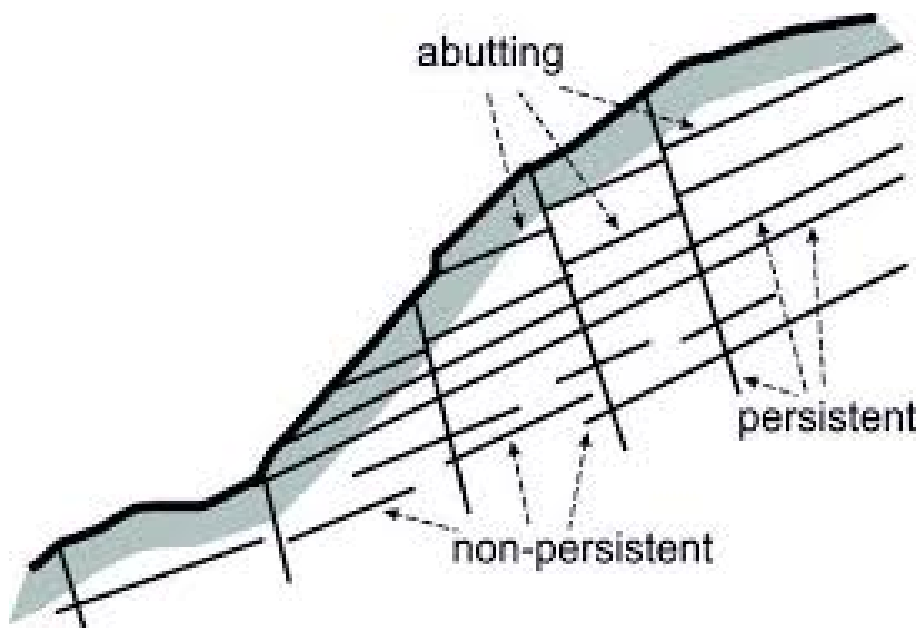
Rock Joint Survey - Important geomechanical properties of discontinuities

- Orientation
- Spacing
- **Persistence**
- Roughness
- Aperture
- Filling
- Wall strength
- Seepage
- Number of sets
- Block size



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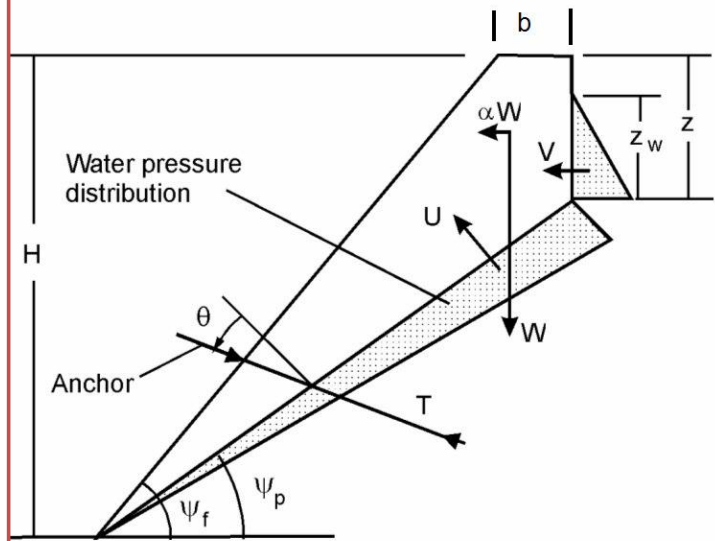
Joint Persistence



Plane Failure Analysis based on water-filled critical tension crack depth

Other forces:

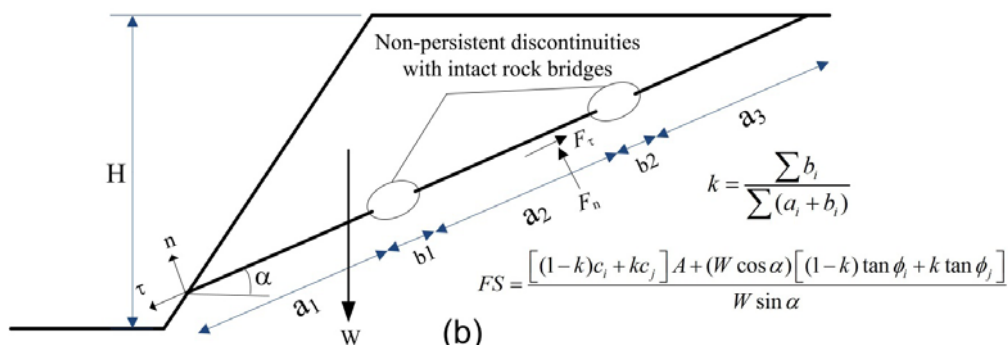
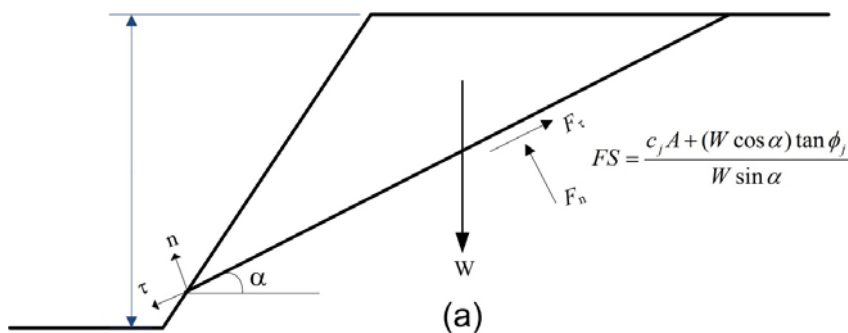
- Surcharge on top of slope
- Ground water (pushing and reduction in normal force)
- Reinforcement (tension, load capacity)
- Earthquakes (horizontal acceleration)



Hoek: Practical Rock Engineering

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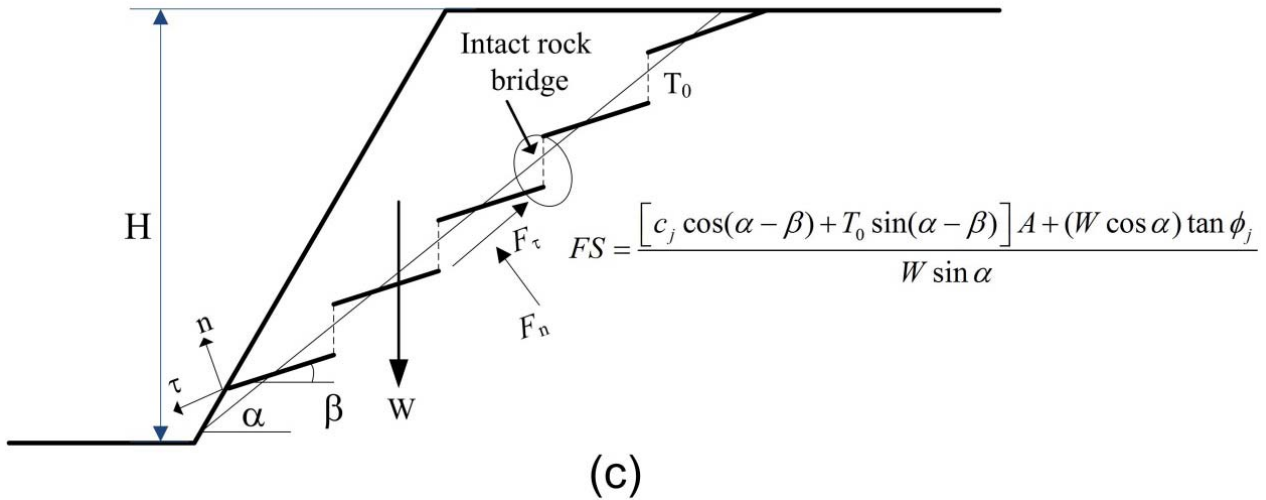
Analytical approach (a, b)



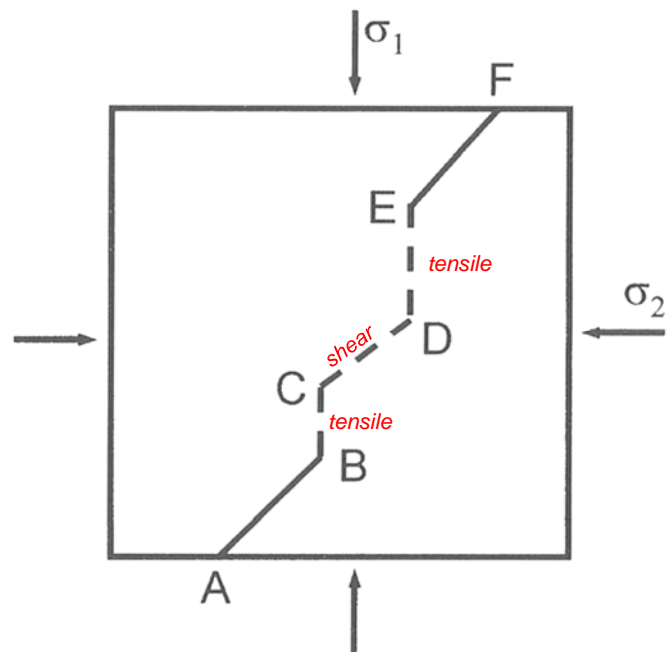
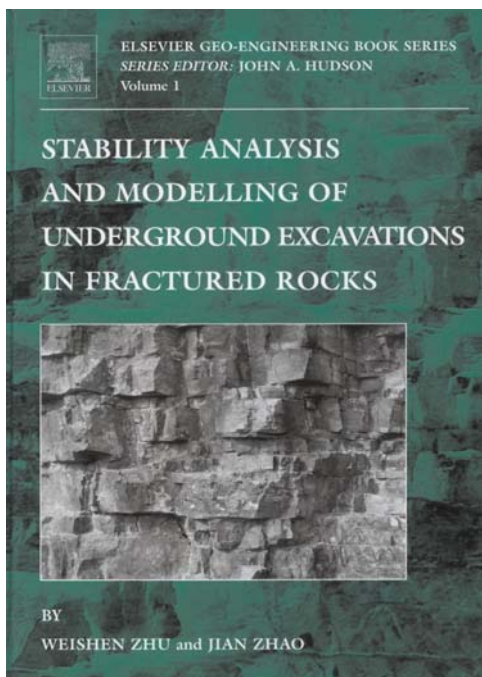
Jennings, J.E. (1970) A Mathematical theory for the calculation of the stability of slopes in open cast mines. in Planning of Open Pit Mines, Proceedings, Johannesburg; 87-102.

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Analytical approach (c)

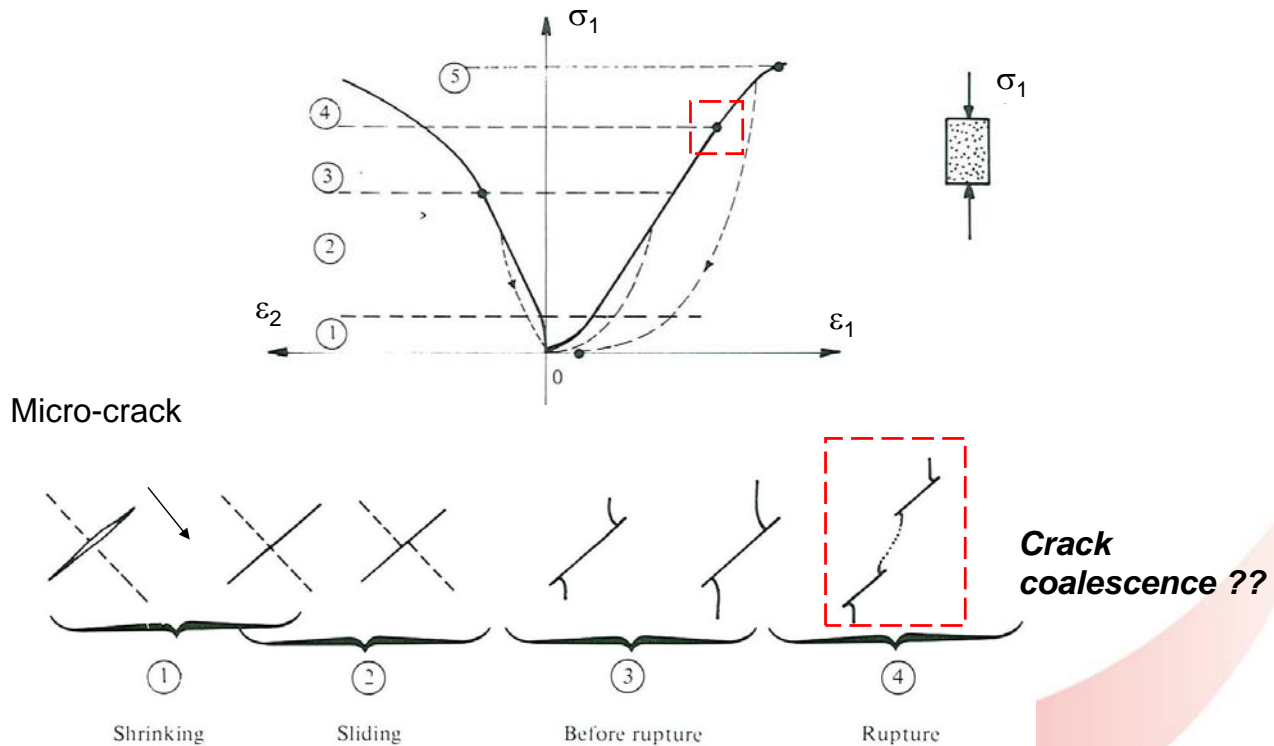


Jaeger, J.C. (1971) *Friction of rocks and stability of rock slopes*. Geotechnique; **21**:97-134.



Encheleon cracking trajectory

Typical Stress-Strain Curve in Rock (lab scale)



Habib, P. (1983) *An outline of soil and rock mechanics*, Cambridge University Press.

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Fracture Mechanics Approach

Two scenarios

- Initiation of new cracks in **intact material**
- Propagation from existing **crack tip**

Let's see the mechanics

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Fracture Mechanics Approach

new crack initiation in intact material

- For shear crack $\frac{\sigma_1 - \sigma_3}{2} = c_i \cos \phi_i + \frac{\sigma_1 + \sigma_3}{2} \sin \phi_i, \quad \sigma_3 > \sigma_t$
- For tensile crack $\sigma_3 = \sigma_t$

where

- c_i and ϕ_i are the cohesion and friction angle of the intact material
- σ_1 and σ_3 are the maximum principal stress and the minimum principal stress, respectively

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Fracture Mechanics Approach

Propagation from existing crack tip

$$\cos \frac{\theta_0}{2} \left(K_I \cos^2 \frac{\theta_0}{2} - \frac{3}{2} K_{II} \sin \theta_0 \right) = K_{IC}$$

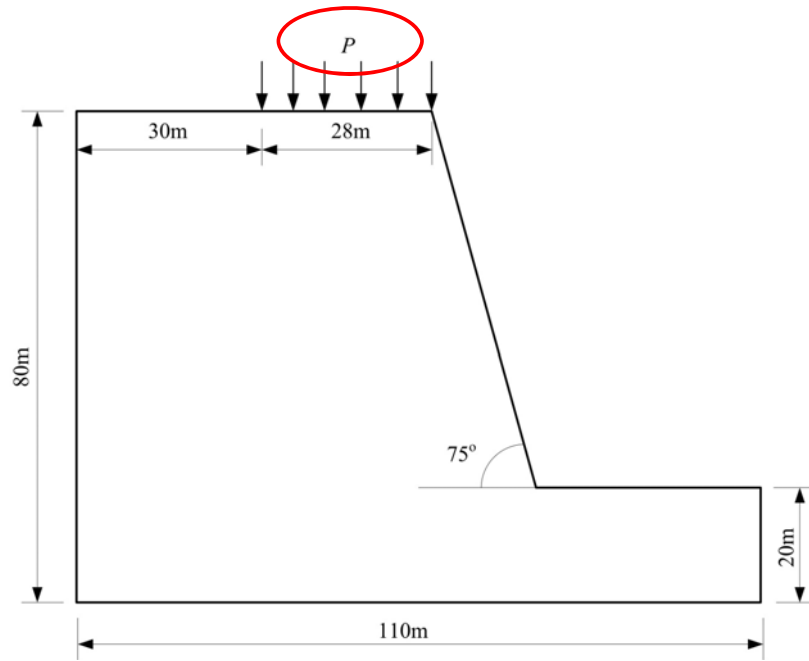
where

- K_I and K_{II} are the mode I and mode II stress intensity factors respectively
- K_{IC} is the mode I fracture toughness and the propagation angle θ_0 can be obtained by the following equation:

$$\theta_0 = \begin{cases} 2 \arctan \left(\frac{|K_I| - \sqrt{K_I^2 + 8K_{II}^2}}{4K_{II}} \right) & K_{II} = 0 \\ 0 & K_{II} \neq 0 \end{cases}$$

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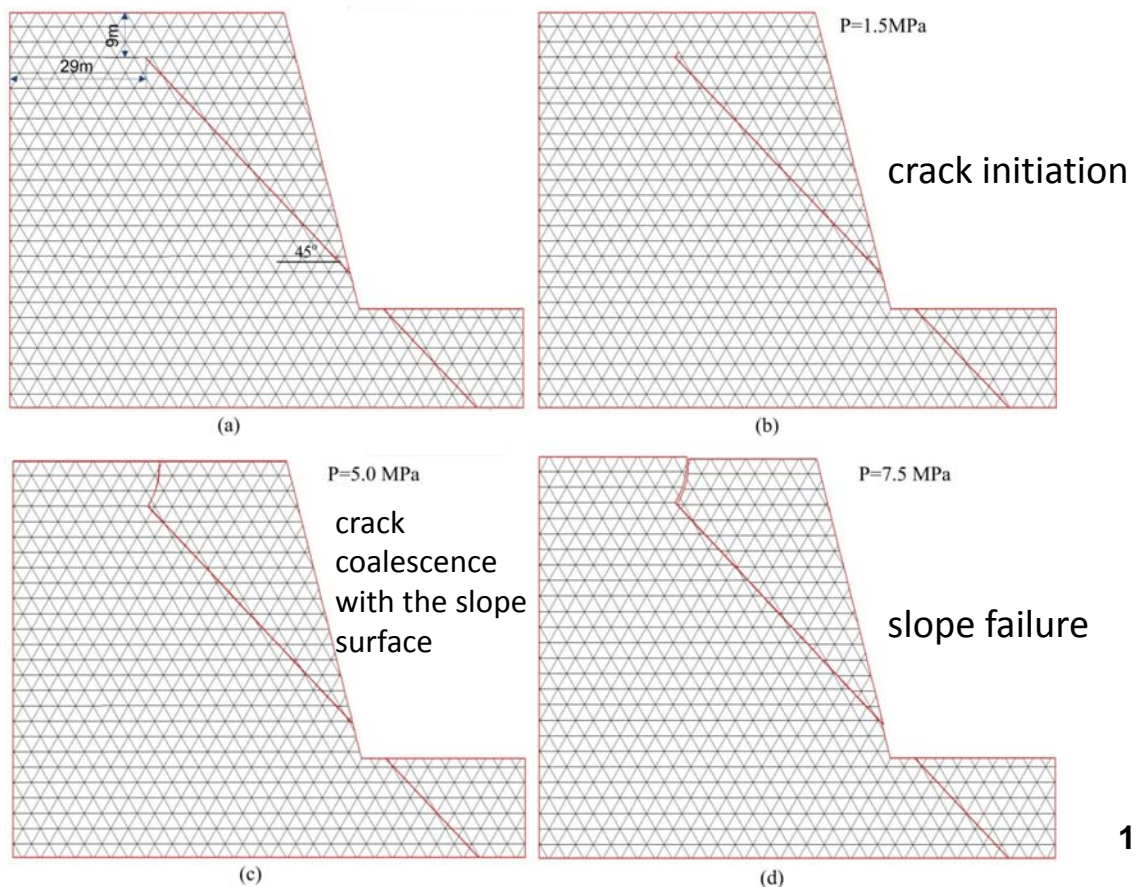
Slope geometry



Intact						Discontinuity		
Density ρ (kg/m ³)	Young's modulus E (GPa)	Poisson's ratio ν	Tensile strength σ_t (MPa)	Friction angle ϕ_i (°)	Cohesion c_i (MPa)	Friction Angle ϕ_j (°)	Cohesion c_j (MPa)	Fracture toughness K_{Ic} (MPa·m ^{1/2})
3100	33.7	0.15	13.1	23.5	18.1	33	0-0.1	1.0

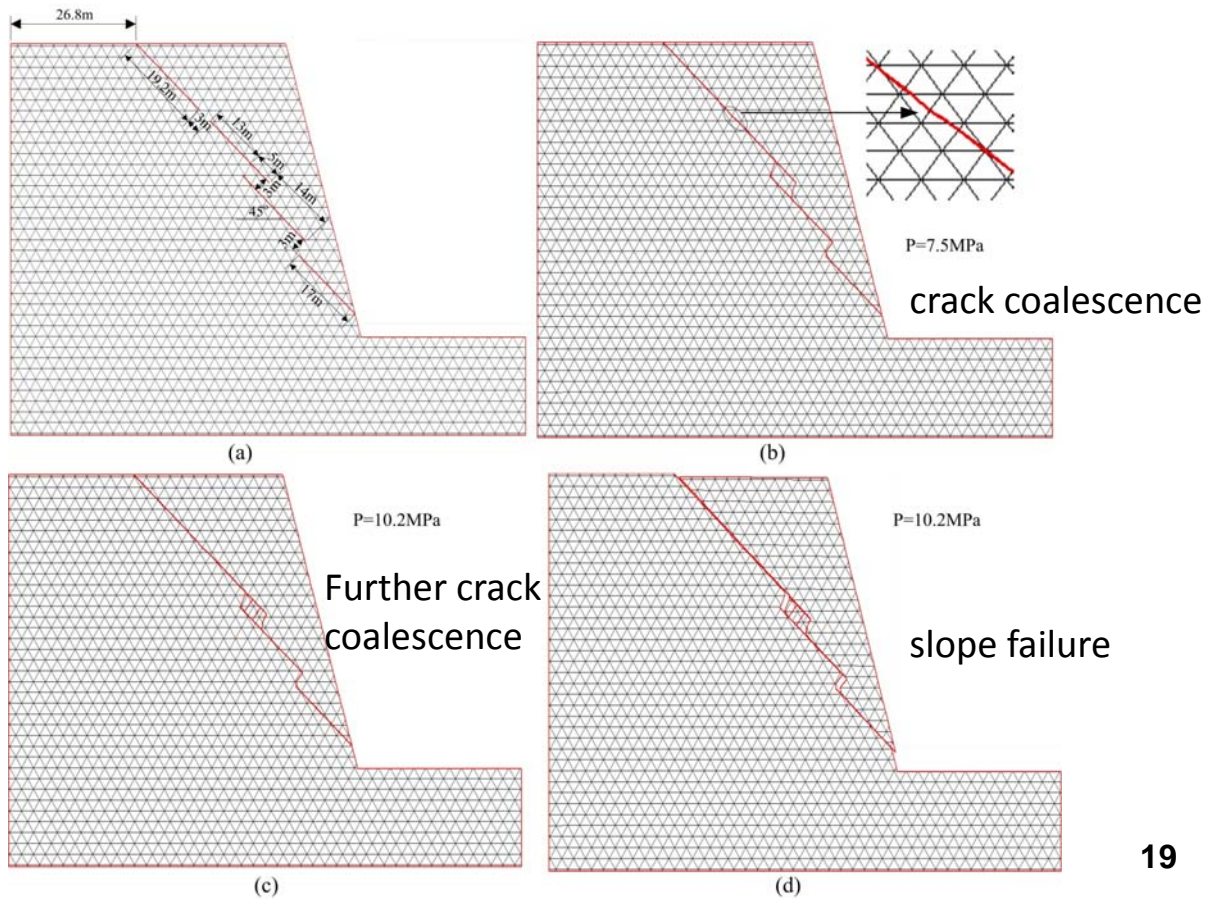
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slope containing one pre-existing discontinuity



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slope containing four pre-existing discontinuities



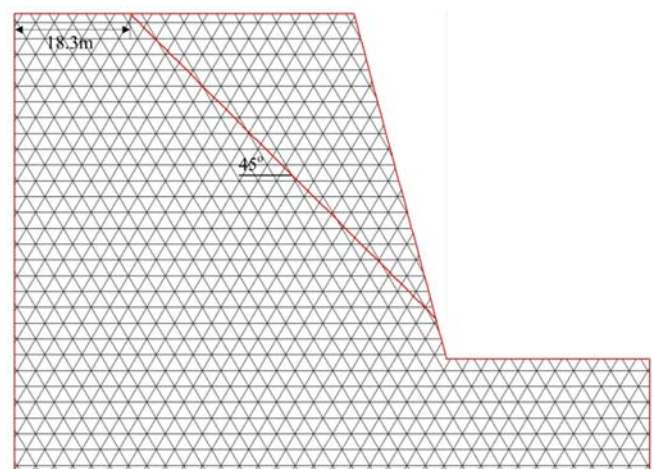
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Factor of Safety (SF)

- SF calculation is based on successively increasing the **acceleration of gravity** until the slope fails.

$$SF = \frac{g_{trial}}{g_0}$$

- g_0 = acceleration of gravity in the initial state
- g_{trial} = acceleration of gravity at failure



Li, L.C., C.A. Tang, W.C. Zhu, and Z.Z. Liang (2009) *Numerical analysis of slope stability based on the gravity increase method*. *Computers and Geotechnics*; **36**:1246-1258.

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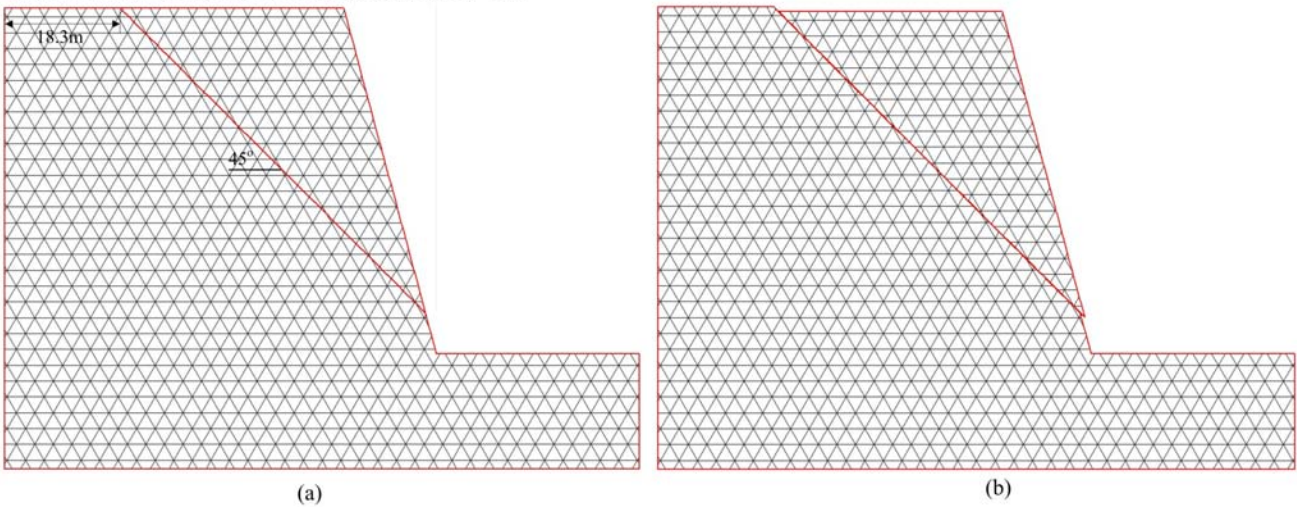
Factor of Safety (SF) – continuous discontinuity

Example 1

SF(Jennings)=5.03

SF(NMM with displacement-dependent method)=5.12

SF(NMM without displacement-dependent method)=4.85



Displacement-dependent cohesion reduction method (Wang et al. 2013)

- cohesion between the discontinuity surfaces **degrades** with increasing movement along the surfaces.
- enables **relative movements** between contact pairs and the removal of the cohesion based on the accumulated relative sliding

Wang, L.Z., et al. (2013) *Development of discontinuous deformation analysis with displacement-dependent interface shear strength*. Computers and Geotechnics; **47**:91-101.

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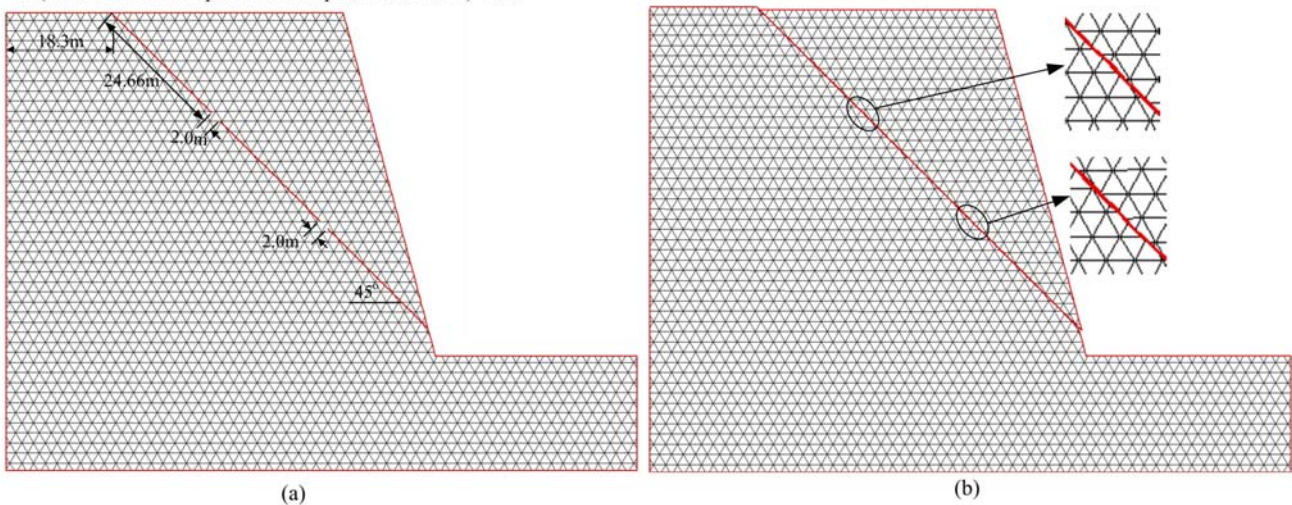
Factor of Safety (SF) – coplanar non-persistent discontinuity

Example 2

SF(Jennings)=6.32

SF(NMM with displacement-dependent method)=5.65

SF(NMM without displacement-dependent method)=5.38



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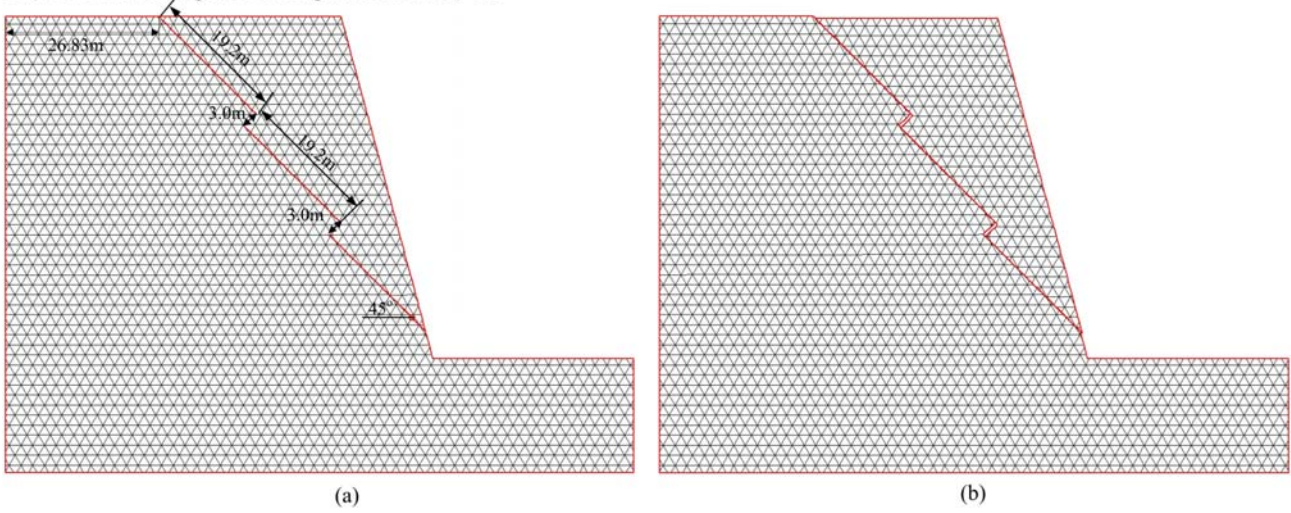
Factor of Safety (SF) – stepped non-persistent discontinuity

Example 5

SF(Jaeger)=10.7

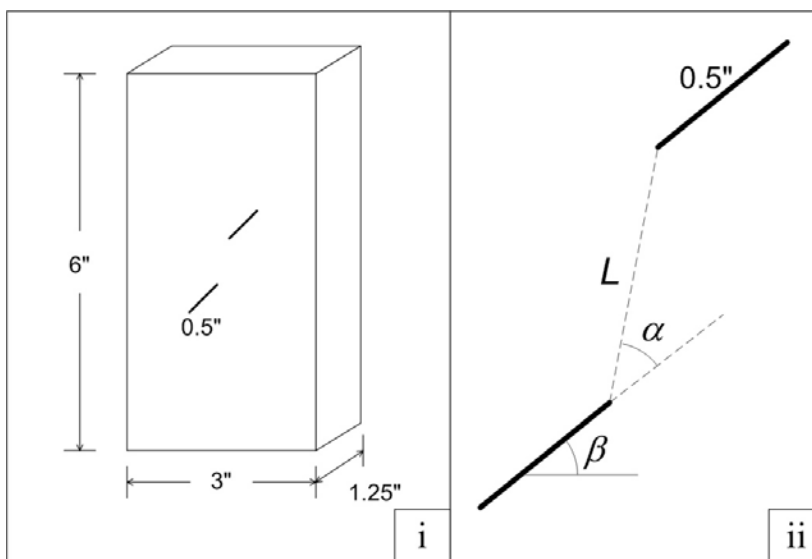
SF(NMM with displacement-dependent method)=9.8

SF(NMM without displacement-dependent method)=9.3



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Experimental study



Geometry representation

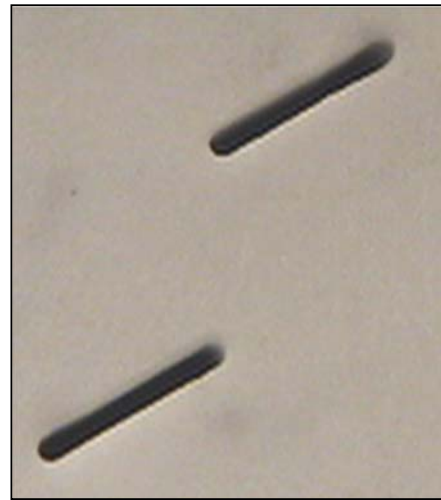
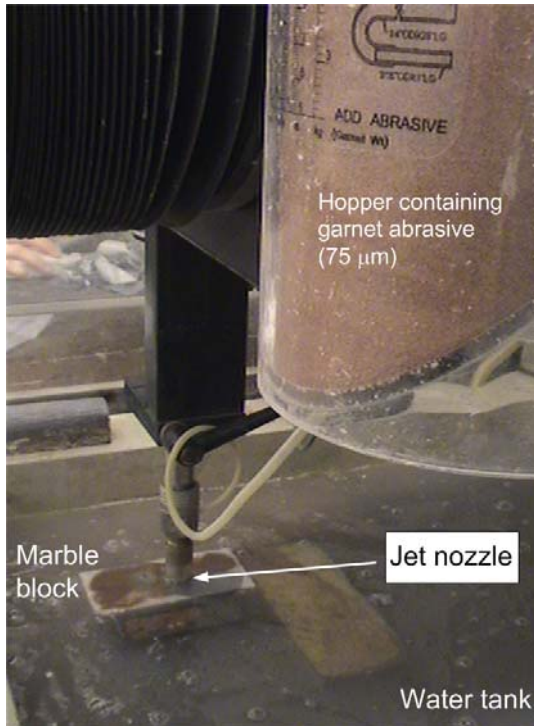
$L - \beta - \alpha$, e.g. 2a-30-45

a = half flaw length

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Water Abrasive Jet

Carrara Marble

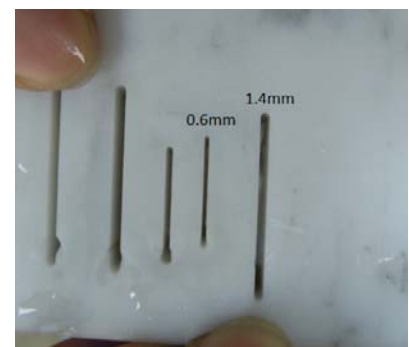


Flaw length
13mm (0.5")

Flaw aperture
1.3mm (0.05")

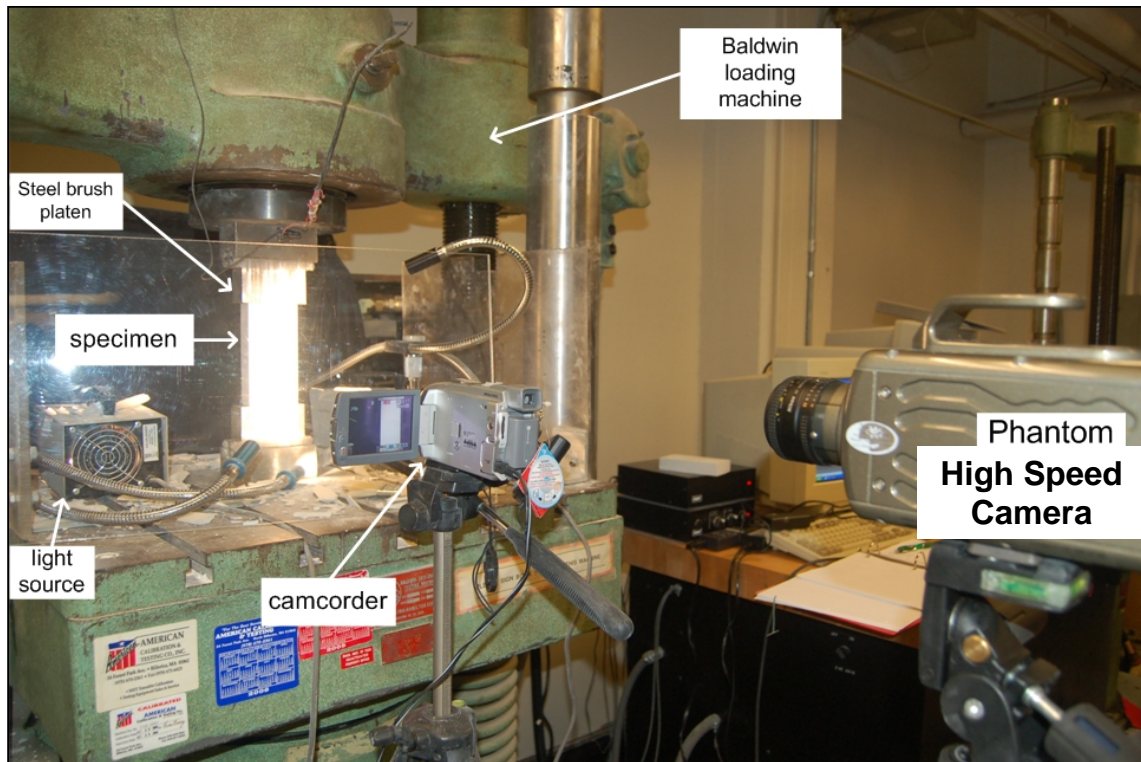
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OMAX water abrasive jet



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Uniaxial Compression Loading Test

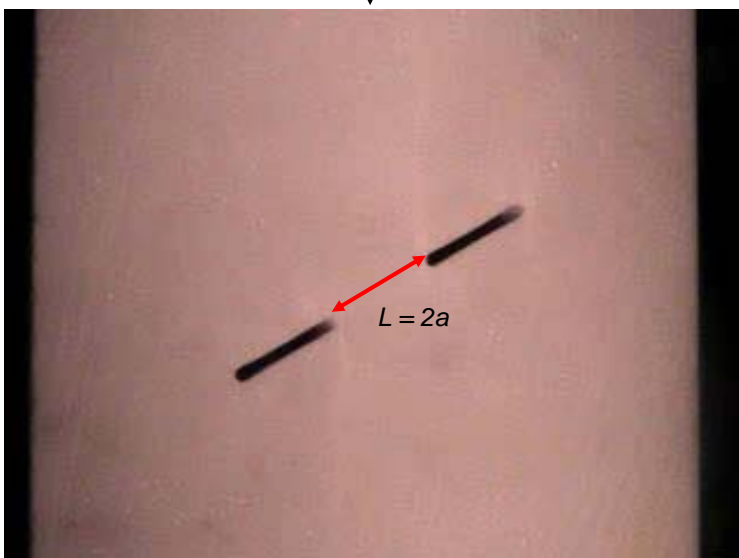


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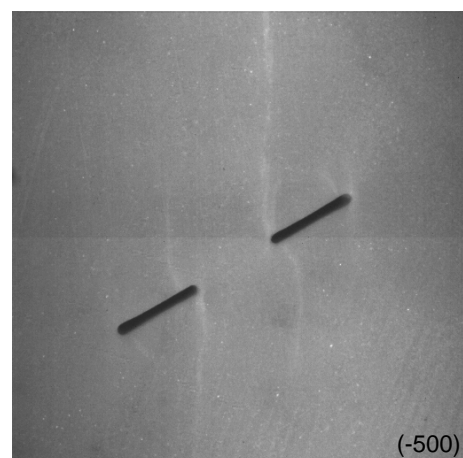
Normal Speed vs High Speed videos

Normal speed video

$\downarrow \sigma_1$



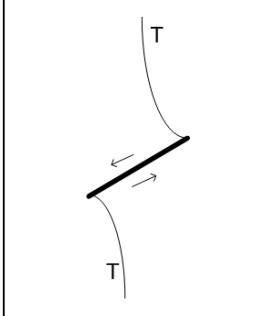
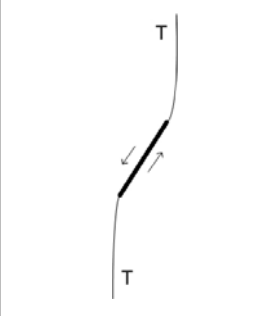
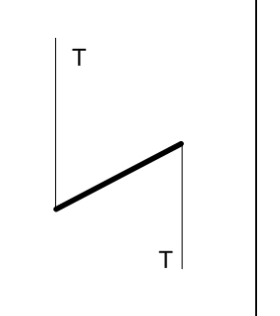
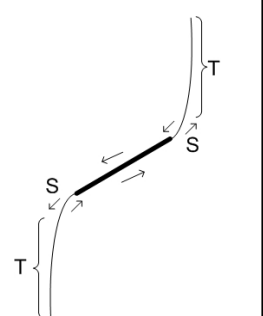
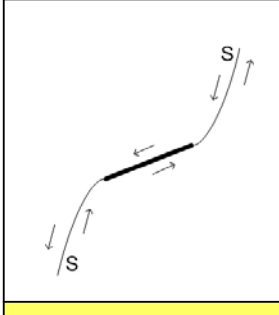
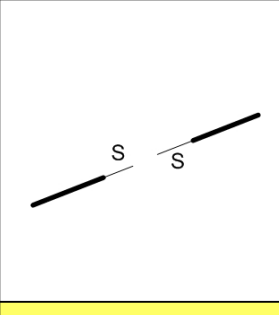
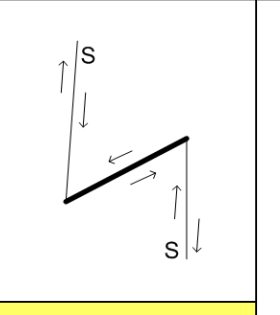
High speed video



- 2000 frames per second
- Video played at 45 times slower

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Results - generalized crack types

			
Type 1 <i>tensile</i> crack (tensile wing crack)	Type 2 <i>tensile</i> crack	Type 3 <i>tensile</i> crack	Mixed <i>tensile-shear</i> crack
			
Type 1 <i>shear</i> crack	Type 2 <i>shear</i> crack	Type 3 <i>shear</i> crack	

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Next Step ...

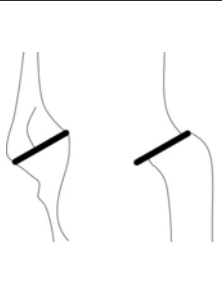
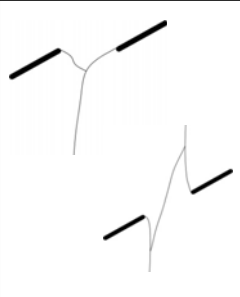
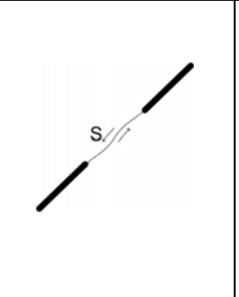
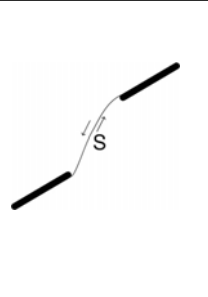
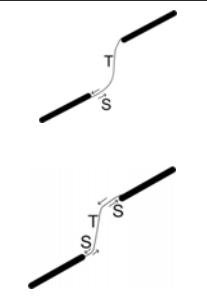
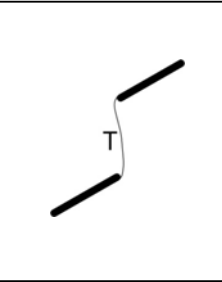
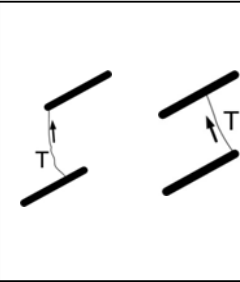
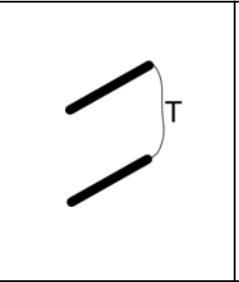
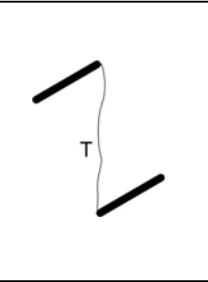
Fundamental Crack Types



Coalescence Patterns

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Results - coalescence categories

				
(1) No coalescence	(2) Indirect coalescence	(3) Type 2 shear crack(s)	(4) Type 1 shear crack(s)	(5) Mixed shear-tensile crack(s)
				(3) – (9) Direct coalescence
(6) Type 2 tensile crack(s)	(7) Type 1 tensile cracks	(8) Tensile crack (type not assigned)	(9) Type 3 tensile cracks	

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Conclusions

- **Progressive** rock slope failure requires the consideration of two factors
 - **slide plane** development
 - internal **rock mass** deformation/degradation

Fracture mechanics approach

- Crack tip
- Intact rock

Conventional approach

- Assumed to be persistent
- Failed when tensile strength or shear strength is overcome

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Thank you

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